Time : 3 hours] ***[Total Marks : 100***

Note : *(i) Attempt All the problems. The choice of problems is internal as indicated.*

Q. 1. Attempt any *Four* parts of the following : — **$5 \times 4 = 20$**

(a) A 4 kg object falls from rest at time $t = 0$ in a medium offering a resistance in kg numerically equal to twice its instantaneous velocity in m/sec. Find the velocity and distance travelled at any time $t > 0$, and also the limiting velocity.

Ans. Let v m/sec be the velocity of the object.

Accelerating force $= (4 - 2v)$ N

The equation of motion is

$$\frac{4}{g}\frac{dv}{dt} = 4 - 2v$$

$$\frac{2dv}{4-2v} = \frac{g}{2}dt$$

$$-\log(4-2v) = \frac{g}{2}t + c_1 \qquad \ldots(1)$$

when $t = 0, v = 0 \quad \Rightarrow \quad c_1 = -\log 4$

$$(1) \Rightarrow \quad -\log(4-2v) = \frac{gt}{2} - \log 4$$

$$\log\left(\frac{4}{4-2v}\right) = \frac{gt}{2}$$

$$\frac{4}{4-2v} = e^{gt/2}$$

$$\frac{4-2v}{4} = e^{-gt/2}$$

$$4 - 2v = 4e^{-gt/2}$$

$$v = 2(1 - e^{-gt/2}) \quad \ldots(2)$$

which gives the velocity of the object.

The limiting velocity is the velocity when $t \to \infty$ from (2), the limiting velocity $= 2$

Now from (2) $\rightarrow$

$$\frac{dx}{dt} = 2(1 - e^{-gt/2})$$

$$dx = 2(1 - e^{-gt/2})dt$$

Integrating, $x = 2\left(t + \frac{2}{g}e^{-gt/2}\right) + c_2 \; \ldots(3)$

Initially when $t = 0, x = 0$

$$c_2 = -\frac{4}{g}$$

$$(3) \Rightarrow \qquad x = 2\left(t + \frac{2}{g}e^{-gt/2}\right) - \frac{4}{g}$$

$$x = 2t - \frac{4}{g}(1 - e^{-gt/2})$$

Q. 1. (b) Solve

$$\mathbf{(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0}$$

Ans. $(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0 \qquad \ldots(1)$

Multiplying by $x^h y^k$, we have

$(3x^h y^{k+1} - 2x^{h+1}y^{k+3})dx +$

$(4x^{h+1}y^k - 3x^{h+2}y^{k+2})dy = 0$

For this equation to be exact we must have

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$3(k+1)x^h y^k - 2(k+3)x^{h+1}y^{k+2}$

$= 4(h+1)x^h y^k - 3(h+2)x^{h+1}y^{k+2}$

which holds when

$3(k+1) = 4(h+1)$ and $-2(k+3)$

$= -3(h+2)$

solving these equations, we have

$h = 2, k = 3$

I.F. $= x^2y^3$

Multiplying (1) by x^2y^3, we have

$(3x^2y^4 - 2x^3y^6)dx$

which is exact

The solution is

$$\int_{y\ \text{constant}} (3x^2y^4 - 2x^3y^6)dx = c$$

$$x^3y^4 - \frac{x^4}{2}y^6 = c$$

Q. 1. (c) solve

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 2y = e^x$$

Ans. $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 2y = e^x$

$$(D^3 - D^2 - 4D - 2)y = e^x$$

A.E is $D^3 - D^2 - 4D - 2 = 0$

$$(D+1)(D^2 - 2D - 2) = 0$$

$$D = -1, \frac{2 \pm \sqrt{4+8}}{2}$$

$$D = -1, 1 \pm i\sqrt{3}$$

$$\text{C.F.} = c_1e^{-x} + e^x(c_2 \cos x\sqrt{3} + c_3 \sin x\sqrt{3})$$

$$\text{P.I.} = \frac{1}{D^3 - D^2 - 4D - 2}e^x$$

$$= \frac{1}{1 - 1 - 4 - 2}e^x$$

$$= -\frac{e^x}{6}$$

$$y = \text{C.F} + \text{P.I.}$$

$$y = c_1e^{-x} + e^x(c_2 \cos x\sqrt{3} + c_3 \sin x\sqrt{3}) - \frac{e^x}{6}$$

Q. 1. (d) solve $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 4 \cot 2x$

Ans. $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 4 \cot 2x$

A.E. is $D^3 + 4D = 0$

$$D(D^2 + 4) = 0$$

$$D = 0, \pm 2i$$

$$\text{C.F} = c_1e^{0.x} + e^{0.x}(c_2 \cos 2x + c_3 \sin 2x)$$

$$= c_1 + c_2 \cos 2x + c_3 \sin 2x$$

$$\text{P.I.} = \frac{1}{D^3 + 4D} 4 \cot 2x$$

$$= \frac{1}{\quad} 4 \cot 2x$$

$$= \frac{1}{D^2 + 4}\frac{4}{2} \log \sin 2x$$

$$= 2\frac{1}{D^2 + 4} \log \sin 2x$$

$$= \frac{2}{4i}\left[\frac{1}{D - 2i} - \frac{1}{D + 2i}\right] \log \sin 2x$$

$$= \frac{1}{2i}\left[\frac{1}{D - 2i} - \frac{1}{D + 2i}\right] \log \sin 2x \quad \text{...(1)}$$

$$= \frac{1}{2i}[P_1 - P_2]$$

$$P_1 = \frac{1}{D - 2i} \log \sin 2x$$

$$= e^{2ix}\int e^{-2ix} \log \sin 2x dx$$

$$= e^{2ix}\left[\log \sin 2x\left(\frac{-e^{-2ix}}{2i}\right) - \int \frac{2 \cos 2x}{\sin 2x}\left(\frac{-e^{-2ix}}{2i}\right)dx\right]$$

$$= -\frac{1}{2i} \log \sin 2x + \frac{2e^{2ix}}{2i} \int \frac{\cos 2x}{\sin 2x}(\cos 2x - i \sin 2x)dx$$

$$= -\frac{1}{2i} \log \sin 2x + \frac{2e^{2ix}}{2i} \int \left(\frac{\cos^2 2x}{\sin 2x} - i \cos 2x\right)dx$$

$$= -\frac{1}{2i} \log \sin 2x + \frac{2e^{2ix}}{2i} \int (\operatorname{cosec} 2x - \sin 2x - i \cos 2x)dx$$

$$= -\frac{1}{2i} \log \sin 2x + \frac{2e^{2ix}}{2i}\left[\frac{1}{2}\log(\operatorname{cosec} 2x - \cot 2x) + \frac{\cos 2x}{2} - i\frac{\sin 2x}{2}\right]$$

$$= -\frac{1}{2i} \log \sin 2x + \frac{e^{2ix}}{2i}[\log(\operatorname{cosec} 2x - \cot 2x) + \cos 2x - i \sin 2x] \quad \text{...(2)}$$

$$P_2 = \frac{1}{D+2i}\log \sin 2x$$

$$= e^{-2ix}\int e^{2ix}\log \sin 2x \, dx$$

$$= e^{-2ix}\left[\log \sin 2x\left(\frac{e^{2ix}}{2i}\right) - \int \frac{2\cos 2x}{\sin 2x}\left(\frac{e^{2ix}}{2i}\right)dx\right]$$

$$= \frac{1}{2i}\log \sin 2x - \frac{2e^{-2ix}}{2i}$$

$$\int (\operatorname{cosec} 2x - \sin 2x + i\cos 2x)dx$$

$$= \frac{1}{2i}\log \sin 2x - \frac{e^{-2ix}}{2i}$$

$$[\log(\operatorname{cosec} 2x - \cot 2x) + \cos 2x + i\sin 2x] \quad \ldots(3)$$

$$\text{P.I.} = \frac{1}{2i}\left(\frac{-2}{2i}\right)\log \sin 2x$$

$$+\frac{1}{2i}\left(\frac{e^{2ix}+e^{-2ix}}{2i}\right)\log(\operatorname{cosec} 2x - \cot 2x)$$

$$+\frac{1}{2i}\left(\frac{e^{2ix}+e^{-2ix}}{2i}\right)\cos 2x$$

$$-\frac{i}{2i}\left(\frac{e^{2ix}-e^{-2ix}}{2i}\right)\sin 2x$$

$$= \frac{1}{2}\log \sin 2x - \frac{1}{2}\cos 2x \log$$

$$(\operatorname{cosec} 2x - \cot 2x) - \frac{1}{2}\cos^2 2x - \frac{1}{2}\sin^2 2x$$

$$= \frac{1}{2}\log \sin 2x - \frac{\cos 2x}{2}$$

$$\log(\operatorname{cosec} 2x - \cot 2x) - \frac{1}{2}$$

$$y = \text{C.F} + \text{P.I.}$$

$$= c_1 + c_2\cos 2x + c_3\sin 2x -$$

$$\frac{1}{2}(1 - \log \sin 2x) - \frac{\cos 2x}{2}\log$$

$$(\operatorname{cosec} 2x - \cot 2x)$$

Q. 1 (b) Solve $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = e^{-t}$,

Ans. $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = e^{-t}$,

$$\frac{d^2y}{dt^2} - 4\frac{dx}{dt} + 3y = \sin 2t$$

$$\Rightarrow \quad (D^2+3)x + Dy = e^t \quad \ldots(1)$$

$$-4Dx + (D^2+3)y = \sin 2t \quad \ldots(2)$$

Eliminating y from (1) & (2), we get

$$[(D^2+3)(D^2+3) + 4D^2]x$$

$$= (D^2+3)e^t - D\sin 2t$$

$$(D^4 + 6D^2 + 9 + 4D^2)x$$

$$= e^t + 3e^t - 2\cos 2t$$

$$(D^4 + 10D^2 + 9)x = 4e^t - 2\cos 2t$$

A.E. is $D^4 + 10D^2 + 9 = 0$

$$(D^2+9)(D^2+1) = 0$$

$$D = \pm 3i, \pm i$$

C.F $= c_1\cos t + c_2\sin t + c_3\cos 3t + c_4\sin 3t$

$$\text{P.I.} = \frac{1}{D^4 + 10D^2 + 9}(4e^t - 2\cos 2t)$$

$$= \frac{1}{D^4 + 10D^2 + 9}4e^t - \frac{1}{(D^2+9)(D^2+1)}2\cos 2t$$

$$= 4\frac{1}{1+10+9}e^t - 2\frac{1}{(-4+9)(-4+1)}\cos 2t$$

$$= \frac{4}{20}e^t + \frac{2}{15}\cos 2t$$

$$= \frac{e^t}{5} + \frac{2}{15}\cos 2t$$

$$x = c_1\cos t + c_2\sin t + c_3\cos 3t + c_4\sin 3t + \frac{e^t}{5} + \frac{2}{15}\cos 2t \quad \ldots(3)$$

$$\frac{dx}{dt} = -c_1\sin t + c_2\cos t - 3c_3\sin 3t + 3c_4\cos 3t + \frac{e^t}{5} - \frac{4}{15}\sin 2t$$

$$\frac{d^2x}{dt^2} = -c_1\cos t - c_2\sin t - 9c_3\cos 3t - 9c_4\sin 3t + \frac{e^t}{5} - \frac{8}{15}\cos 2t$$

$$\frac{dy}{dt} = e^t - \frac{d^2x}{dt^2} - 3x$$

$$= e^t + c_1 \cos t + c_2 \sin t + 9c_3 \cos 3t + 9c_4 \sin 3t - \frac{e^t}{5} + \frac{8}{15}\cos 2t - 3c_1 \cos t - 3c_2 \sin t - 3c_3 \cos 3t - 3c_4 \sin 3t - \frac{3e^t}{5} - \frac{6}{15}\cos 2t$$

$$\frac{dy}{dt} = -2c_1 \cos t - 2c_2 \sin t + 6c_3 \cos 3t + 6c_4 \sin 3t - \frac{e^t}{5} + \frac{2}{15}\cos 2t$$

$$y = -2c_1 \sin t + 2c_2 \cos t + 6c_3 \frac{\sin 3t}{3} - 6c_4 \frac{\cos 3t}{3} - \frac{e^t}{5} + \frac{2}{15}\cdot\frac{\sin 2t}{2}$$

$$y = -2c_1 \sin t + 2c_2 \cos t + 2c_3 \sin 3t - 2c_4 \cos 3t - \frac{e^t}{5} + \frac{\sin 2t}{15} \quad \ldots(4)$$

Equations (3) & (4) give the required solutions

Q. 1. (f) Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - \lambda^2 y = 0$

Ans. $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - \lambda^2 y = 0$

Put $x = e^z, z = \log x,\ D = \frac{d}{dz}$

$[D(D-1) + D - \lambda^2]y = 0$

$(D^2 - \lambda^2)y = 0$

A.E. is $D^2 - \lambda^2 = 0$

$D^2 = \lambda^2 \Rightarrow D = \pm\lambda$

$y = C.F. = c_1 e^{\lambda z} + c_2 e^{-\lambda z} = c_1 x^{\lambda} + c_2 x^{-\lambda}$

Q. 2. Attempt any *Four* parts of the following : — **$5 \times 4 = 20$**

(a) Find the Laplace transform of

$$\begin{cases} t^2 & 0 < t < 2 \\ t-1 & 2 < t < 3 \end{cases}$$

Ans. $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ t-1 & 2 < t < 3 \\ 7 & t > 3 \end{cases}$

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\bar{f}(s) = \int_0^2 e^{-st} t^2\,dt + \int_2^3 e^{-st}(t-1)\,dt + \int_3^\infty e^{-st}\,7\,dt$$

$$= \left\{t^2\left(\frac{e^{-st}}{-s}\right) - (2t)\left(+\frac{e^{-st}}{s^2}\right) + (2)\left(-\frac{e^{-st}}{s^3}\right)\right\}_0^2 + \left\{(t-1)\left(-\frac{e^{-st}}{s}\right) - \left(\frac{e^{-st}}{s^2}\right)\right\}_2^3 + 7\left(-\frac{e^{-st}}{s}\right)_3^\infty$$

$$= \left(-\frac{4}{s}e^{-2s} - \frac{4}{s^2}e^{-2s} - \frac{2}{s^3}e^{-2s} + \frac{2}{s^3}\right) + \left(-\frac{2}{s}e^{-3s} - \frac{e^{-3s}}{s^2} + \frac{e^{-2s}}{s} + \frac{e^{-2s}}{s^2}\right) + 7\left(0 + \frac{e^{-3s}}{s}\right)$$

$$= \left(-\frac{4}{s} - \frac{4}{s^2} - \frac{2}{s^3} + \frac{1}{s} + \frac{1}{s^2}\right)e^{-2s} + \left(-\frac{2}{s} - \frac{1}{s^2} + \frac{7}{s}\right)e^{-3s} + \frac{2}{s^3}$$

$$= \left(-\frac{3}{s} - \frac{3}{s^2} - \frac{2}{s^3}\right)e^{-2s} + \left(\frac{5}{s} - \frac{1}{s^2}\right)e^{-3s} + \frac{2}{s^3}$$

$$\bar{f}(s) = \frac{2}{s^3} - \left(\frac{3}{s} + \frac{3}{s^2} + \frac{2}{s^3}\right)e^{-2s} + \left(\frac{5}{s} - \frac{1}{s^2}\right)e^{-3s}$$

Q. 2. (b) If $L\{f(t)\} = F(s)$, show that $L\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty F(s)\,ds$. **Hence, find the**

$$f(t) = \int_0^t \frac{\sin \tau}{\tau} d\tau$$

Ans. $L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)dt$

Integrating both sides w.r.to s from 0 to ∞

$$\int_s^\infty F(s)ds = \int_s^\infty \left(\int_0^\infty e^{-st} f(t)dt \right) ds$$

$$= \int_0^\infty \int_s^\infty f(t)e^{-st} ds dt$$

on changing the order of integration

$$= \int_0^\infty f(t) \left(\int_s^\infty e^{-st} ds \right) dt$$

$$= \int_0^\infty f(t) \left(-\frac{e^{-st}}{t} \right)_s^\infty dt$$

$$= \int_0^\infty e^{-st} \frac{f(t)}{t} dt$$

$$= L\left\{ \frac{f(t)}{t} \right\}$$

$$L\left\{ \frac{f(t)}{t} \right\} = \int_s^\infty F(s)ds$$

$$L\{\sin t\} = \frac{1}{s^2+1} = F(s)$$

$$L\left\{ \frac{\sin t}{\tau} \right\} = \int_s^\infty \frac{1}{s^2+1} ds = [\tan^{-1} s]_s^\infty$$

$$= \tan^{-1} \infty - \tan^{-1} s$$

$$= \frac{\pi}{2} - \tan^{-1} s = F_1(s)$$

$$L\left\{ \int_0^t \frac{\sin \tau}{\tau} d\tau \right\} = \frac{F_1(s)}{s} = \frac{1}{s}\left(\frac{\pi}{2} - \tan^{-1} s \right)$$

Q. 2. (c) Find the function whose Laplace transform is $\ln\left(1 + \frac{1}{s}\right)$

Ans. Let

$$f(t) = L^{-1}\left\{ \log\left(1 + \frac{1}{s}\right) \right\} = L^{-1}\left\{ \log\left(\frac{s+1}{s} \right) \right\}$$

$$f(t) = L^{-1}\{\log(s+1) - \log s\}$$

$$tf(t) = L^{-1}\left[-\frac{d}{ds}\{\log(s+1) - \log s\} \right]$$

$$= L^{-1}\left[-\frac{1}{s+1} + \frac{1}{s} \right]$$

$$tf(t) = -e^{-t} + 1$$

$$f(t) = \frac{1 - e^{-t}}{t}$$

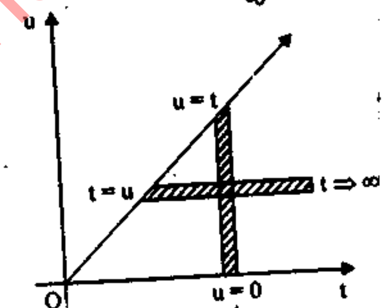
Q. 2. (d) State and prove convolution theorem for Laplace transform.

Ans. Convolution Theorem :

If $L^{-1}\{F(s)\} = f(t)$ and

$L^{-1}\{G(s)\} = g(t)$

then $L^{-1}\{F(s)\,G(s)\} = \int_0^t f(u)g(t-u)du$



Proff : Let $\phi(t) = \int_0^t f(u)\, g(t-u)du$

$$L\{\phi(t)\} = \int_0^\infty e^{-st} \left\{ \int_0^t f(u)\, g(t-u)du \right\} dt$$

$$= \int_0^\infty \int_0^t e^{-st} f(u)\, g(t-u)du dt \quad \ldots(1)$$

On changing the order of integration, we get,

$$L\{\phi(t)\} = \int_0^\infty \int_u^\infty e^{-st} f(u) g(t-u)\, dt du$$

$$= \int_0^\infty e^{-su} f(u) \left\{ \int_0^\infty e^{-s(t-u)} g(t-u)dt \right\} du$$

$$= \int_0^\infty e^{-su} f(u) \left\{ \int_0^\infty e^{-sv} g(v)dv \right\} du$$

putting $t - u = v$

$$= \int_0^\infty e^{-su} f(u)\, G(s)\, du = \left(\int_0^\infty e^{-su} f(u)du \right) G(s)$$

$$= F(s).G(s)$$

$$L^{-1}\{F(s)\,G(s)\} = \phi(t) = \int_0^t f(u)g(t-u)du.$$

Q. 2. (e) Solve for $y(t)$ the equation

$$y(t) = 1 + \int_0^t y(\tau)\cos(t-\tau)d\tau$$

Ans. $y(t) = 1 + \int_0^t y(\tau)\cos(t-\tau)d\tau$

$$y = 1 + y * \cos t$$

Taking Laplace transform, $L\{y\} = Y(s)$

$$Y(s) = \frac{1}{s} + Y(s)\frac{s}{s^2+1}$$

$$Y(s)\left[1 - \frac{s}{s^2+1}\right] = \frac{1}{s}$$

$$Y(s) = \frac{s^2+1}{s(s^2-s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2-s+1}$$

$$A = 1, B = 0, C = 1 \quad Y(s) = \frac{1}{s} + \frac{1}{s^2-s+1}$$

Takaing inverse laplace transform

$$y(t) = L^{-1}\left\{\frac{1}{s}\right\} + L^{-1}\left\{\frac{1}{s^2-s+1}\right\}$$

$$= 1 + L^{-1}\left\{\frac{1}{\left(s-\frac{1}{2}\right)^2 - \frac{3}{4}}\right\}$$

$$= 1 + e^{t/2}\,L^{-1}\left\{\frac{1}{s^2 - \left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= 1 + \frac{e^{t/2}}{\sqrt{3}/2}\sinh\left(\frac{t\sqrt{3}}{2}\right)$$

$$y(t) = 1 + \frac{2}{\sqrt{3}}e^{t/2}\sinh\left(\frac{t\sqrt{3}}{2}\right)$$

Q. 2. (f) Solve using Laplace transform

$y(0) = -2, y'(0) = 8$

Ans. $y''(t) + 4y'(t) + 4y(t) = 6e^{-t}$

$y(0) = -2 \quad y'(0) = 8$

Taking Laplace transform, we get

$$L\{y''\} + 4L\{y'\} + 4\,L\{y\} = 6L\{e^{-t}\}$$

$$s^2L(y) - sy(0) - y'(0) + 4\{sL(y) - y(0)\} + 4L(y) = \frac{6}{s+1}$$

$$L(y)(s^2+4s+4) + 2s - 8 + 8 = \frac{6}{s+1}$$

$$L(y)(s+2)^2 = \frac{6}{s+1} - 2s$$

$$L(y) = \frac{6}{(s+1)(s+2)^2} - \frac{2s}{(s+2)^2}$$

$$L(y) = \frac{6}{s+1} - \frac{6}{s+2} - \frac{6}{(s+2)^2} - \frac{2(s+2-2)}{(s+2)^2}$$

Taking inverse Laplace transform

$$y = 6L^{-1}\left\{\frac{1}{s+1}\right\} - 6\,L^{-1}\left\{\frac{1}{s+2}\right\} - 6L^{-1}\left\{\frac{1}{(s+1)^2}\right\} - 2L^{-1}\left\{\frac{(s+2)-2}{(s+2)^2}\right\}$$

$$= 6e^{-t} - 6e^{-2t} - 6e^{-2t}L^{-1}\left\{\frac{1}{s^2}\right\} - 2e^{-2t}L^{-1}\left\{\frac{s-2}{s^2}\right\}$$

$$= 6e^{-t} - 6e^{-2t} - 6e^{-2t}t - 2e^{-2t}L^{-1}\left\{\frac{1}{s} - \frac{2}{s^2}\right\}$$

$$= 6e^{-t} - 6e^{-2t} - 6te^{-2t} - 2e^{-2t}(1-2t)$$

$$y(t) = 6e^{-t} - 8e^{-2t} - 2te^{-2t}$$

Q. 3. Attempt any *Two* parts of the following : — **$10 \times 2 = 20$**

(a) Expand $f(x) = k, < 0 < x < 2$ in a half range

(1) Sine series

(2) Cosine series

Ans. $f(x) = k, 0 < x < 2$

(i) Half range sine series

$$f(x)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$$

$$\text{where, } b_n=\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{2}dx$$

$$=\frac{2}{2}\int_0^2 k\sin\frac{n\pi x}{2}dx$$

$$=k\left[-\frac{\cos\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right]_0^2$$

$$=\frac{2k}{n\pi}[-\cos n\pi+1]$$

$$=\frac{2k}{n\pi}[1-(-1)^n]$$

$$f(x)=\frac{2k}{\pi}\sum_{n=1}^{\infty}[1-(-1)^n]\sin\frac{n\pi x}{2}$$

(ii) Half range cosine series

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{2\pi x}{2}$$

where

$$a_0=\frac{2}{l}\int_0^l f(x)dx=\frac{2}{2}\int_0^2 kdx=k(x)_0^2$$

$$a_0=2k$$

$$a_n=\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{2}dx$$

$$=\frac{2}{2}\int_0^2 k\cos\frac{n\pi x}{2}dx$$

$$=k\left(\frac{\sin\frac{n\pi x}{2}}{(n\pi/2)}\right)_0^2=\frac{2k}{n\pi}(\sin n\pi-\sin 0)$$

$$=0$$

$$f(x)=\frac{2k}{2}=k$$

Q. 3. (b) If $f(x)=\begin{cases}\pi x & 0<x<1\\ \pi(2-x) & 1<x<2\end{cases}$

using half range cosine series expansion, show that

$$\frac{1}{1}+\frac{1}{3^4}+\frac{1}{5^4}+\dots\dots=\frac{\pi^4}{96}$$

Ans. $f(x)=\begin{cases}\pi x, & 0<x<1\\ \pi(2-x), & 1<x<2\end{cases}$

Half range cosine series

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{2}$$

$$a_0=\frac{2}{2}\int_0^2 f(x)dx=\int_0^1\pi x dx+\int_1^2\pi(2-x)dx$$

$$=\pi\left(\frac{x^2}{2}\right)_0^1+\pi\left(2x-\frac{x^2}{2}\right)_1^2$$

$$=\frac{\pi}{2}+\pi\left[(4-2)-\left(2-\frac{1}{2}\right)\right]$$

$$=\frac{\pi}{2}+\pi\left[2-\frac{3}{2}\right]=\frac{\pi}{2}+\frac{\pi}{2}=\pi$$

$$a_n=\frac{2}{2}\int_0^2 f(x)\cos\frac{n\pi x}{2}dx$$

$$=\int_0^1\pi(x)\cos\frac{n\pi x}{2}dx+\int_1^2\pi(2-x)\cos\frac{n\pi x}{2}dx$$

$$=\pi\left[x\left(\frac{\sin\frac{n\pi x}{2}}{n\pi/2}\right)-\left(-\frac{\cos\frac{n\pi x}{2}}{n^2\pi^2/4}\right)\right]_0^1+\pi\left[(2-x)\left(\frac{\sin\frac{n\pi x}{2}}{n\pi/2}\right)-(-1)\left(-\frac{\cos\frac{n\pi x}{2}}{n^2\pi^2/4}\right)\right]_1^2$$

$$=\pi\left[\left(\frac{2}{n\pi}\sin\frac{n\pi}{2}+\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}\right)-\left(\frac{4}{n^2\pi^2}\right)\right]$$

$$+\pi\left[-\frac{4}{n^2\pi^2}\cos n\pi-\left\{\frac{2}{n\pi}\sin\frac{n\pi}{2}-\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}\right]$$

$$=\pi\left[\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}-\frac{4}{n^2\pi^2}-\frac{4}{n^2\pi^2}\cos n\pi\right.$$

$$-\frac{2}{n\pi}\sin\frac{n\pi}{2}+\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}\Bigg]$$

$$=\pi\Bigg[\frac{2}{n\pi}\sin\frac{n\pi}{2}+\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}-\frac{4}{n^2\pi^2}$$

$$-\frac{4}{n^2\pi^2}\cos n\pi-\frac{2}{n\pi}\sin\frac{n\pi}{2}+\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}\Bigg]$$

$$=\frac{4\pi}{n^2\pi^2}\left[2\cos\frac{n\pi}{2}-1-\cos n\pi\right]$$

$$=\frac{4}{n^2\pi}\left[2\cos\frac{n\pi}{2}-1-\cos n\pi\right]$$

when n is odd $\cos\frac{n\pi}{2}=0$ and $\cos n\pi=-1$

$a_n=0 \Rightarrow a_1=a_3=a_5 \ldots.=0$

When n is even

$$a_2=\frac{4}{2^2\pi}[2\cos\pi-1-\cos 2\pi]=-\frac{16}{2^2\pi}$$

$$a_4=\frac{4}{4^2\pi}[2\cos 2\pi-1-\cos 4\pi]=0$$

$$a_6=\frac{4}{6^2\pi}[2\cos 3\pi-1-\cos 6\pi]=\frac{-16}{6^2\pi}$$

$$f(x)=\frac{\pi}{2}-\frac{16}{\pi}\left(\frac{1}{2^2}\cos\frac{2\pi x}{2}+\frac{1}{6^2}\cos\frac{6\pi x}{2}+\ldots\ldots\right)$$

$$f(x)=\frac{\pi}{2}-\frac{16}{\pi}\left(\frac{1}{2^2}\cos\pi x+\frac{1}{6^2}\cos 3\pi x+\ldots\right)$$

$$a_0^2=\pi^2,\ a_2^2=\frac{(16)^2}{2^4\pi^2},\ a_4^2=\frac{(16)^2}{6^4\pi^2}\ldots..$$

The root mean square value.

$$(\bar{y})^2=\frac{1}{2}\int_0^2 y^2dx$$

$$(\bar{y})^2=\frac{1}{2}\left[\int_0^1\pi^2x^2dx+\int_1^2\pi^2(2-x)^2dx\right]$$

$$=\frac{\pi^2}{2}\left[\left(\frac{x^3}{3}\right)_0^1+\int_1^2(4+x^2-4x)dx\right]$$

$$=\frac{\pi^2}{2}\left[\frac{1}{3}+\left(4x+\frac{x^3}{3}-4\frac{x^2}{2}\right)_1^2\right]$$

$$=\frac{\pi^2}{2}\left[\frac{1}{3}+\left(8+\frac{8}{3}-8\right)-\left(4+\frac{1}{3}-2\right)\right]$$

$$=\frac{\pi^2}{2}\left[\frac{1}{3}+\frac{8}{3}-2-\frac{1}{3}\right]=\frac{\pi^2}{2}\left(\frac{2}{3}\right)$$

$$=\frac{\pi^2}{3}$$

$$\frac{\pi^2}{3}=\frac{2}{22}\left[\frac{a_0^2}{2}+a_1^2+a_2^2+a_3^2+a_4^2+\ldots.\right]$$

$$\frac{2\pi^2}{3}=\frac{\pi^2}{2}+\frac{(16)^2}{2^4\pi^2}+\frac{(16)^2}{6^4\pi^2}+\frac{(16)^2}{10^4\pi^2}\ldots.$$

$$\frac{2\pi^2}{3}-\frac{\pi^2}{2}=\frac{(16)^2}{2^4\pi^2}\left(1+\frac{1}{3^4}+\frac{1}{5^4}\ldots\ldots\right)$$

$$\frac{\pi^2}{6}=\frac{(16)^2}{2^4\pi^2}\left(1+\frac{1}{3^4}+\frac{1}{5^4}\ldots..\right)$$

$$\frac{\pi^2}{6}\times\frac{\pi^2}{16}=1+\frac{1}{3^4}+\frac{1}{5^4}\ldots..$$

$$\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}\ldots..$$

Q. 3. (c) Solve

$$\frac{\partial^3u}{\partial x^3}-3\frac{\partial^3u}{\partial x^2\partial y}+4\frac{\partial^3u}{\partial y^3}=e^{x+2y}$$

Ans. $\frac{\partial^3u}{\partial x^3}-3\frac{\partial^3u}{\partial x^2\partial y}+4\frac{\partial^3u}{\partial y^3}=e^{x+2y}$

$(D^3-3D^2D'+4D'^3)u=e^{x+2y}$

A. E is $m^3-3m^2+4=0$

$m=-1, 2, 2$

C.F $=f_1(y-x)+f_2(y+2x)+xf_3(y+2x)$

$$\text{P.I.}=\frac{1}{D^3-3D^2D'+4D'^3}e^{x+2y}$$

$D=1, D'=2$

$$=\frac{1}{1-3\times2+4\times8}e^{x+2y}$$

$$=\frac{1}{27}e^{x+2y}$$

$u = C.F + P.I$
$= f_1(y - x) + f_2(y + 2x) + xf_3(y + 2x) + \frac{e^{x+2y}}{27}$

Q. 4. Attempt any *Two* parts of the following : — $10 \times 2 = 20$

(a) Find the solution (s) using the power series method $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2y = 0$.

Ans. $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2y = 0$ ...(1)

Here $x = 0$ is a regular singular point.

Let $y = \sum a_m x^{m+k}$ ...(2)

$$\frac{dy}{dx} = \sum a_m(m+k)x^{m+k-1}$$

$$\frac{d^2y}{dx^2} = \sum a_m(m+k)(m+k-1)x^{m+k-2}$$

$$x\sum a_m(m+k)(m+k-1)x^{m+k-2} + \sum a_m(m+k)x^{m+k-1} + x^2\sum a_m x^{m+k} = 0$$

$$\sum a_m[(m+k)(m+k-1) + (m+k)]x^{m+k-1} + \sum a_m x^{m+k+2} = 0$$

$$\sum a_m(m+k)^2 x^{m+k-1} + \sum a_m x^{m+k+2} = 0 \quad ...(3)$$

Equating to zero the coefficient of x^{k-1}

$a_0k^2 = 0 \Rightarrow k^2 = 0$

$k = 0, 0$

Equating to zero the coefficient of x^k

$a_1(k+1)^2 = 0 \Rightarrow a_1 = 0$

Equating to zero the coefficient of x^{k+1}

$a_2(k+2)^2 = 0 \Rightarrow a_2 = 0$

Equating to zero the coefficient of x^{m+k+2}

$$\boxed{a_{m+2} = -\frac{a_m}{(m+k+3)^2}} \quad ...(4)$$

$$a_3 = -\frac{1}{(k+3)^2}a_0$$

$$a_4 = -\frac{1}{(k+4)^2}a_1 = 0$$

$$a_4 = a_7 = a_{10} = 0$$

$$a_5 = -\frac{1}{(k+5)^2}a_2 = 0, a_8 = 0, a_{11} = 0$$

$$a_6 = -\frac{1}{(k+6)^2}a_3 = \frac{1}{(k+3)^2(K+6)^2}a_0$$

$$a_9 = -\frac{1}{(k+9)^2}a_6 = -\frac{1}{(k+3)^2(k+6)^2(k+9)^2}a_0$$

$$y = x^k a_0\left[1 - \frac{x^3}{(k+3)^2} + \frac{x^6}{(k+3)^2(k+6)^2} - \frac{x^9}{(k+3)^2(k+6)^2(k+9)^2} \ldots\right] \quad ...(5)$$

when $k = 0$

$$y_1 = a_0\left[1 - \frac{x^3}{3^2} + \frac{x^6}{3^2 6^2} - \frac{x^9}{3^2 6^2 9^2} \ldots\right] \quad ..(6)$$

Differentiating (5) w.r.t. k

$$\frac{\partial y}{\partial k} = (x^k \log x)a_0\left[1 - \frac{x^3}{(k+3)^2} + \frac{x^6}{(k+3)^2(k+6)^2} - \frac{x^9}{(k+3)^2(k+6)^2(k+9)^2} \ldots\right]$$

$$+ x^k a_0\left[\frac{2x^3}{(k+3)^3} - \frac{2x^6}{(k+3)^3(k+6)^2} - \frac{2x^6}{(k+3)^2(k+6)^3} \ldots\right]$$

putting $k = 0$ $\left(\frac{\partial y}{\partial k}\right)_{k=0} = y_2$

$$= (\log x)a_0\left[1 - \frac{x^3}{3^2} + \frac{x^6}{3^2 6^2} - \frac{x^9}{3^2 6^2 9^2} \ldots\right] + a_0\left[\frac{2x^3}{3^3} - \frac{2x^6}{3^3 6^2} - \frac{2x^6}{3^2 6^3} \ldots\right] \ldots (7)$$

Hence the general solution

$y = c_1 y_1 + c_2 y_2$

Q. 4. (b) Establish any tow of the following recurrence formulae for the Legendre polynomials

(1) $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$

(2) $(x^2-1)P_n'(x) = n\{xP_n(x) - p_{n-1}(x)\}$

(3) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

Ans. (1) $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$

Let $v = (x^2-1)^n$

$$\frac{dv}{dx} = n(x^2-1)^{n-1} 2x$$

$$(x^2-1)\frac{dv}{dx} = n(x^2-1)^n 2x = 2nxv$$

$$(1-x^2)\frac{dv}{dx} + 2nxv = 0$$

Differentiating $(n+1)$ times by Leibnitz theorem,

$$\left[(1-x^2)\frac{d^{n+2}v}{dx^{n+2}} + (n+1)(-2x)\frac{d^{n+1}v}{dx^{n+1}} + \frac{(n+1)n}{2!}(-2)\frac{d^n v}{dx^n}\right] + 2n\left[x\frac{d^{n+1}v}{dx^{n+1}} + (n+1)\frac{d^n v}{dx^n}\right] = 0$$

$$(1-x^2)\frac{d^{n+2}v}{dx^{n+2}} - 2x\frac{d^{n+1}v}{dx^{n+1}} + n(n+1)\frac{d^n v}{dx^n} = 0$$

$$(1-x^2)\frac{d^2 v_n}{dx^2} - 2x\frac{dv_n}{dx} + n(n+1)v_n = 0$$

Where $v_n = \frac{d^n v}{dx^n}$

which is Legendre's equation and v_n is its solution.

$v_n = \frac{d^n}{dx^n}(x^2-1)^n$ contains only positive power of x, it must be a constant multiple of $P_n(x)$

$v_n = cP_n(x)$

$$cP_n(x) = \frac{d^n}{dx^n}(x^2-1)^n = \frac{d^n}{dx^n}[(x-1)^n(x+1)^n] \quad ...(1)$$

$$= (x-1)^n n! + n_{c_1} n(x-1)^{n-1}\frac{d^{n-1}}{dx^{n-1}}(x+1)^n + ... + (x+1)^n n!$$

$= n!(x+1)^n +$ terms containing powers of $(x-1)$

Putting $x = 1$

$cP_n(1) = n!\,2^n \Rightarrow c = 2^n n!$

$$(1) \Rightarrow P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

(2) $(x^2-1)P_n'(x) = n\{xP_n(x) - p_{n-1}(x)\}$

Rec. formula $P_n' - xP_{n-1}' = nP_{n-1}$ ...(1)

Rec. formula $xP_n' - P_{n-1}' = nP_n$ ...(2)

Multiplying (2) by x and subtracting from (1), we get

$$(1-x^2)P_n' = n[P_{n-1} - xP_n]$$

$$(x^2-1)P_n' = n[xP_n - P_{n-1}]$$

(3) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

$$(1-2xh+h^2)^{-1/2} = \sum_{n=0}^{\infty} h^n P_n(x) \quad ...(1)$$

Differentiating both sides w.r.t h, we get

$$-\frac{1}{2}(1-2xh+h^2)^{-3/2}(2h-2x) = \sum_{n=0}^{\infty} nh^{n-1}P_n(x)$$

$$(x-h)(1-2xh+h^2)^{-1/2} = (1-2xh+h^2)\sum nh^{n-1}P_n(x)$$

$$(x-h)\sum h^n P_n(x) = (1-2xh+h^2)\sum nh^{n-1}P_n(x)$$

Equating the coefficient of h^n in both sides.

$$xP_n - P_{n-1} = (n+1)P_{n+1} - 2nxP_n + (n-1)P_{n-1}$$

$(n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}$

Q. 4. (c) Prove any two of the following recurrence formulae for the Bessel's functions $J_n(x)$:

(1) $2n J_n(x) = x(J_{n+1} + j_{n-1})$

(2) $J_{n+3}(x) + J_{n+5}(x) = \frac{2}{x}(n + 4)$

$J_{n+4}(x)$

(3) $x^2 J_r''(x) = (n^3 - n - x^2) J_n(x)$

$+ xJ(x)_{n+1}$

Ans. (1) $2n J_n(x) = x(J_{n+1} + J_{n-1})$

$$\frac{d}{dx}(x^n J_n) = x^n J_{n-1}$$

$$x^n J_n' + nx^{n-1} J_n = x^n J_{p-1}$$

Dividing by x^n

$$J_n' + \frac{n}{x} J_n = J_{n-1} \quad ...(1)$$

Again

$$\frac{d}{dx}(x^{-n} J_n) = -x^{-n} J_{n+1}$$

$$x^{-n} J_n' + nx^{-n-1} J_n = -x^{-n} J_{n+1}$$

or $-J_n^1 + \frac{n}{x} J_n = J_{n+1}$ ...(2)

Adding (1) & (2), we get,

$$\frac{2n}{x} J_n = J_{n+1} + J_{n-1}$$

$$2nJ_n = x(J_{n+1} + J_{n-1})$$

(2) $J_{n+3}(x) + J_{n+5}(x) = \frac{2}{x}(n + 4)$

$J_{n+4}(x)$

$$J_n' + \frac{n}{x} J_n = J_{n-1}$$

Put $n = n + 4$

$$J_{n+4}' + \frac{(n+4)}{x} J_{n+4} = J_{n+3} \quad ...(1)$$

Again $-J_n' + \frac{n}{x} J_n = J_{n+1}$

Put $n = n + 4$

$$-J_{n+4}' + \frac{(n+4)}{x} J_{n+4} = J_{n+5} \quad ...(2)$$

Adding (1) & (2) we get

$$\frac{2(n+4)}{x} J_{n+4} = J_{n+5} + J_{n+3}$$

(3) $x^2 J_n''(x) = (n^3 - n - x^2) J_n(x)$

$+ xJ(x)_{n+1}$

$$xJ_n' = nJ_n - xJ_{n+1} \quad ...(1)$$

Differentiating

$$xJ_n'' + J_n' = nJ_n' - xJ_{n+1}' - J_{n+1}$$

$$x^2 J_n'' = (n-1)xJ_n' - x^2 J_{n+1}' - xJ_{n+1} \quad ...(2)$$

Rec. from $xJn' = -nJ_n + xJ_{n-1}$

$$xJ_{n+1}' = -(n+1)J_{n+1} + xJ_n \quad ...(3)$$

$(2) \Rightarrow x^2 J_n'' = (n-1) xJ_n'[nJ_n - xJ_{n+1}]$

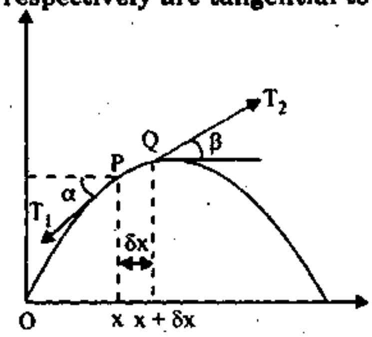
$-x[-(n+1)J_{n+1} + xJ_n - xJ_{n+1}]$

$x^2 J_n'' = (n^2 - n - x^2)J_n + xJ_{n+1}$

Q. 5. Solve any *Two* parts of the following :— $10 \times 2 = 20$

(a) Derive the one dimensional wave equation for vibrating string under suitable conditions.

Ans. One dimensional wave equation : Let m be the mass per unit length of the string. Consider the motion of an element PQ of length δs, the tension T_1 and T_2 at P and Q respectively are tangential to the curve.



Since is no motion in the horizontal direction, we have

$$T_1 \cos\alpha = T_2 \cos\beta = T \text{ (constant)} \quad ...(1)$$

Mass of element $PQ = m\delta s$

By Newton's second law of motion, the equation of motion

$$m\delta s \frac{\partial^2 y}{\partial t^2} = T_2 \sin\beta - T_1 \sin\alpha$$

$$\frac{m\delta s}{T} \frac{\partial^2 y}{\partial t^2} = \frac{T_2 \sin\beta}{T_2 \cos\beta} - \frac{T_1 \sin\alpha}{T_1 \cos\alpha}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m\delta s}(\tan\beta - \tan\alpha)$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m\partial x}\left[\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

Since $\delta s = \delta x$ to a first approximation and $\tan\alpha$ and $\tan\beta$ are the slopes of the curve of the string at x and $x + \delta x$

$$\text{or } \frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\left[\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\partial x}\right]$$

$$\text{or } \quad = \frac{T}{m}\frac{\partial^2 y}{\partial x^2} \text{ as } \partial x \to 0$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}} \quad \text{where } c^2 = \frac{T}{m}$$

The boundary conditions

(i) $y = 0$ when $x = 0$ (ii) $y = 0$ when $x = l$

If the string is made to vibrate by pulling it into a curve $y = f(x)$ and then relasing it with the velocity $g(x)$, the initial conditions are

(i) $y = f(x)$ when $t = 0$

(ii) $\left(\frac{\partial y}{\partial t}\right) = g(x)$ when $t = 0$

Q. 5. (b) Describe the method of separation of variables for solving a partial differential equation. Hence solve the one dimensional heat equation.

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

under suitable initial and boundary conditions.

Ans. Method of Separation of variables : In this method, we take solution which breaks up into a product of functions each of which contains only one of the independent variable.

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t} \text{ or } \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

Let $u = X(x)\, T(t)$ ...(2) be the solution of equation (1)

Where X is a function of $x$and T is a function of t only.

$$XT' = c^2 X'' T$$

$$\frac{X''}{X} = \frac{1}{c^2}\frac{T'}{T} = k \quad ...(3)$$

$$\frac{d^2 X}{dx^2} - kX = 0 \text{ and } \frac{dT}{dt} - kc^2 T = 0$$

(i) When k is positive and $= p^2$

$$\frac{d^2 X}{dx^2} - p^2 X = 0 \text{ and } \frac{dT}{dt} - p^2c^2 T = 0$$

$$X = c_1 e^{px} + c_2 e^{-px},\ T = c_3 e^{c^2p^2t}$$

(ii) When k is negative and $= -p^2$

$$\frac{d^2 X}{dx^2} + p^2 X = 0 \text{ and } \frac{dT}{dt} + p^2c^2 T = 0$$

$$X = c_1 \cos px + c_2 \sin px,\ T = c_3 e^{-c^2p^2t}$$

(iii) when $k = 0$

$X = c_1 x + c_2,\ T = c_3$

since u decrease as time t increases, the solution of heat equation in

$$u = (c_1 \cos px + c_2 \sin px) e^{-c^2p^2t}$$

Q. 5. (c) Characterize the following partial differential equations into elliptic, parabolic and hyperbolic equations

$$A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2}$$

$$= F(x, y, u, u_x, u_y)$$

Here A, B, C may be functions of x and y.

Ans. $A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2}$

$= F(x, y, u, u_x, u_y)$ ...(1)

Equation (1) is

(i) Elliptic if $(2B)^2 - 4AC < 0$

$4B^2 - 4AC < 0$

$B^2 - AC < 0$

(ii) Hyperbolic if $(2B)^2 - 4AC > 0$

$B^2 - AC > 0$

(iii) Parabolic if $(2B)^2 - 4AC = 0$

$B^2 - AC = 0$

The differential equations governing the phenomenon of one and two dimensional heat flow are parabolic and elliptic respectively, while the motion of a vibrating string is described by a hyperbolic partial differential equation.